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IEEE Guide for Evaluation of Torque Pulsations During Starting of Synchronous Motors

Sponsor

Electric Machinery Committee
of the
IEEE Power Engineering Society

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Abstract: A uniform method for calculating and measuring torque pulsations that occur during starting of synchronous motors is provided. Synchronous motors, as discussed in this guide, applies to all types of excited synchronous motors, including laminated or solid, salient or nonsalient machines, as well as nonexcited synchronous-reluctance motors.

Keywords: air gap torque, angular acceleration, electromagnetic, motor, pulsation, rotational velocity, salient, speed sensing, starting stator resistance, synchronous, torque, winding volt-second ampere (VSA) method

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Introduction

(This introduction is not part of IEEE Std 1255-2000, IEEE Guide for Evaluation of Torque Pulsations During Starting of Synchronous Motors.)

As the size of large synchronous motors began to increase in the early 1960s to multimegawatt ratings, shaft breakage began to be a major concern, particularly in applications requiring a long starting interval. Torque pulsations resonating with the natural mechanical frequency of the shaft and load were blamed for these breakages, but theoretical as well as experimental techniques for evaluating this phenomenon were lacking at the time. Technical discussions concerning this problem were scheduled at several conferences in the mid 1960s.

This guide for evaluating these torque pulsations is the result of a more than 30-year effort of a Working Group formed under the auspices of the Synchronous Machines Subcommittee of the Power Engineering Society (PES) Electric Machinery Committee, formed sometime around 1968. The work of many participants over the years is acknowledged, including Jack Imbertson, William Flora, K. C. Cooley, Radhey Mathur, Azad Mesrobian, William L. Billington, L. A. Ferrano, Ernest I. Pollard, J. P. Povlock, John F. Szablya, Jay C. White (deceased), and particularly, former chairperson Gurney Godwin.

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1. Overview

1.1 Scope

This guide provides a uniform method for calculating and measuring torque pulsations that occur during starting of synchronous motors. Synchronous motors, as discussed in this guide, applies to all types of excited synchronous motors, including laminated or solid, salient or nonsalient machines, as well as nonexcited synchronous-reluctance motors.

1.2 Purpose

This guide

a) Defines electromagnetic torque pulsations that occur during starting of synchronous motors.

b) Recommends an equivalent circuit that can be used to describe analytically the behavior of a laminated, salient-pole synchronous motor during starting.

c) Provides the definitions and information necessary to properly determine the circuit parameters of the equivalent circuit.

d) Supplies equations, derived from the circuit, that can be used to calculate the magnitude of the torque pulsation that would occur during starting.

e) Presents several alternative techniques for the on-line measurement of torque pulsations during starting.

1.3 Limitations

Although this guide recommends specific calculation methods and testing techniques, it does not suggest nor imply suitability of the methods for all machine designs or operating conditions. In particular, magnetic hysteresis, eddy currents in the stator, changes in stator resistance due to skin effect, and armature tooth saturation could cause significant deviations.

In modern installations, direct current is applied to the field winding by means of slip rings from a static exciter, or by a rotating brushless excitation system, to establish synchronous operation; the field is typically shorted through a starting resistor during the starting phase of operation. In modern excitation systems, the shorting of the exciter is frequently accomplished by means of a crow-bar thyristor. The action of this device produces a dissymmetry in the induced field current, which must be taken into account.

2. Reference

This guide shall be used in conjunction with the following publication. When the following publication is superseded by an approved revision, the revision shall apply.

IEEE Std 115-1995, IEEE Guide: Test Procedures for Synchronous Machines, Part I—Acceptance and Performance Testing, Part II—Procedures for Parameter Determination for Dynamic Analysis.

3. Problem description

The synchronous motor and its connected load can be represented by a torsional system of interconnected springs and masses, with a large number of springs and masses external to the motor itself. As in any mechanical system, resonances occur at the system natural frequencies, which are algebraically related to the spring constants and masses of the system. In many cases, the connected load has sufficient damping or is sufficiently stiff that these resonances are not of concern. Some systems are inherently flexible and have relatively low resonant frequencies. Such a system is sensitive to impressed torques that could excite the system at one of its mechanical resonant frequencies. The problem is especially pronounced in a synchronous motor, since the torque produced typically sweeps the frequency range from zero to twice the line frequency as it accelerates from rest during starting. Overstressed shafts and couplings, abnormal gear tooth wear, loss of interference fits, and other problems can result when a motor and the equipment coupled to it are subjected to excessive torsional oscillation.

Unlike lateral vibrations that can be readily sensed by touch and measured with relatively common instruments, torsional oscillations can exist with considerable amplitudes and yet be undetectable except by special instrumentation. Since torsional oscillations are so difficult to detect and measure, and yet cause disastrous consequences, it is particularly important that torsional stresses be evaluated when synchronous motors are to drive other equipment.

All synchronous machines develop the following three major components of electromagnetic torque during starting:

- A unidirectional or time-averaged component
- An initial transient (decaying) pulsating component
- A persistent (nondecaying) pulsating component

The unidirectional component is the useful component that acts to accelerate the machine, and results from the forward rotating components of the rotor current interacting with the air gap flux in the same manner as a squirrel cage induction machine. The torque definitions given in the IEEE Standard Dictionary of Electrical and Electronics Terms [B8][1] refer to the time-averaged torque. The transient component results from the initial asymmetrical component of inrush current, which interacts with the symmetrical alternating component of air gap flux in the same manner as a squirrel cage induction motor. The resulting frequency of torque pulsation is essentially equal to the line frequency (i.e., 50 Hz or 60 Hz). The steady pulsating component is

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[1] The numbers in brackets correspond to those of the bibliography in Annex C.

the result of asymmetry of the rotor electrical and magnetic circuits. This guide is concerned primarily with the continuous pulsating component of torque that occurs during asynchronous operation due to the effects of rotor asymmetry.

In general, the persistent pulsating component arises from both rotor geometry variations that repeat once per pole pitch or that reappear once for every two poles. The following are the three causes of rotor asymmetry for which the geometry repeats once per pole pitch:

a) Magnetic permeance variations, which are the result of the salient poles being relatively more easily magnetized at the center of the pole and relatively less easily magnetized at the center of the interpolar space.
b) Main field winding, which encircles only the direct axis and not the quadrature axis.
c) Nonuniform bar placement and composition of the amortisseur or damper winding. The amortisseur winding, which consists of short-circuited bars inserted into slots in the rotor, serves to start the motor in much the same manner as do the rotor bars of a conventional squirrel cage induction motor.

These geometric variations result in a torque pulsation at twice the slip frequency (twice the difference between the synchronous speed and the rotor speed multiplied by the number of pole pairs). When the main field winding is excited by direct current, the electromagnetic geometry repeats once for every two poles, and there exists an additional component of continuous torque pulsation at slip frequency. In most cases, the field winding is shorted either across the terminals or through a resistor. If the motor accelerates without the field excitation already applied, the frequency of the pulsating torque then varies nearly linearly from twice the line frequency at standstill to zero frequency at synchronous speed. However, should the field excitation be applied before the synchronizing speed is reached, an additional large component of pulsating torque appears with a frequency one-half of the original component (i.e., at the slip frequency).

One of the functions of the motor field excitation control circuit is to prevent the application of excitation until the synchronizing speed is close to synchronous speed. Another function is to remove the field excitation and reconnect the discharge resistor should the motor pull out of step after synchronism is reached. For motors with brushless excitation, the design of the excitation system attempts to avoid unnecessary or an unexpectedly large amount of torque pulsation. There are a number of different exciter circuits available from motor manufacturers. Experience has indicated that, with proper operation, the inherent amount of slip-frequency torque pulsation during starting is only a fraction of twice the slip-frequency torque pulsation, and is, therefore, not critical. Hence, this guide is concerned primarily with the starting torque pulsation at twice the slip frequency and not with the premature application of field excitation.

The magnitude of the pulsating torque is essentially determined by the degree of asymmetry of the rotor circuit. A rotor structure that has no significant asymmetry will not have a significant torque pulsation during asynchronous operation. Although the magnitude of twice the slip-frequency pulsating torque varies depending upon the rotor design parameters, experience indicates it is not practical to eliminate this pulsation. Furthermore, the reductions in the magnitude of the pulsating torque that are theoretically possible often can not be obtained in practice because it is necessary to meet other requirements of the drive.

In addition to the torque pulsation, asymmetry of the rotor typically causes a loss in average torque above half speed. This reduction in average torque is the result of the backward traveling component of air gap flux interacting with the stator circuit and creating a braking torque. The greater the degree of asymmetry in the rotor, the larger the amount of torque pulsation and the greater the loss in average torque for operation above one-half synchronous speed. A rotor structure that is extremely asymmetrical, such as a machine having no effective winding on one axis, will develop extreme torque pulsations and will not accelerate beyond one-half synchronous speed. A solid pole design most nearly approximates the single winding case when pole tip connections are not used.

While the factors that affect excitation of most modes of torsional oscillation are primarily contained in the design of the synchronous motor, the design of the external equipment to which it is mechanically connected must also be considered. When the manufacturer of the driven equipment makes a torsional analysis of the complete assembly, the motor manufacturer should be consulted for the motor design data for speed ranges of critical interest that affect torsional vibration. The manufacturer of the driven equipment, in turn, should advise the motor manufacturer of the results of this torsional analysis to permit the design of the motor rotor to be satisfactorily completed. When the driven machine starting requirements are not severe, the motor designer may have some latitude to adjust the motor torque characteristic. An increase in damping to avoid excessive torque pulsations or a shift of the mechanical resonance can improve the drive.

4. Synchronous motor equivalent circuit

The analysis of synchronous motors is generally based on the two-axis (*d-q*) theory developed by R. H. Park. The framework of differential equations, called “Park’s Equations,” forms the basis of most analyses of synchronous machines (see Park [B15]). Using the motor convention for current flow and the notation and terminology of Concordia [B2], Park’s equations are written in a reference frame rotating synchronously with the rotor and in per unit form (see Harris et al. [B6]) as shown in Equation (1) through Equation (6)

$$e_d = p\psi_d - \psi_q p\theta + r i_d \tag{1}$$

$$e_q = p\psi_q + \psi_d\, p\theta + r i_q \tag{2}$$

$$0 = p\psi_{1d} + r_{1d} i_{1d} \tag{3}$$

$$e_{fd} = p\psi_{fd} + r_{fd} i_{fd} \tag{4}$$

$$0 = p\psi_{1q} + r_{1q} i_{1q} \tag{5}$$

$$T_e = \psi_d i_q - \psi_q i_d \tag{6}$$

where

e_d	is the instantaneous value of the armature (i.e., stator) direct axis voltage, in per unit
e_{fd}	is the instantaneous value of the armature quadrature axis voltage, in per unit
e_q	is the instantaneous value of field voltage, in per unit
i_d	is the instantaneous value of the armature direct axis current, in per unit
i_q	is the instantaneous value of the armature quadrature axis voltage, in per unit
i_{1d}	is the instantaneous value of direct axis amortisseur winding current, in per unit
i_{1q}	is the instantaneous value of quadrature axis amortisseur winding current, in per unit
i_{fd}	is the instantaneous value of field current, in per unit
p	is the derivative operator $d/d(\omega t)$, where ω is the base frequency and t is time
r	is the per unit armature resistance per phase (including external resistance)
r_{1d}	is the per unit direct axis amortisseur resistance (referred to the stator winding)
r_{1q}	is the per unit quadrature axis amortisseur resistance (referred to the stator winding)
r_{fd}	is the per unit field resistance (including discharge resistance during starting and referred to the stator winding)

T_e	is the instantaneous electromagnetic torque, in per unit
ψ_d	is the instantaneous armature direct axis flux linkages, in per unit
ψ_q	is theinstantaneous armature quadrature axis flux linkages, in per unit
ψ_{1d}	is the instantaneous direct axis amortisseur winding flux linkages, in per unit
ψ_{1q}	is the instantaneous quadrature axis amortisseur winding flux linkages, in per unit
ψ_{fd}	is the instantaneous field flux linkages, in per unit
p	is the derivative operator $d/d(\omega t)$, where ω is the base frequency and t is time
$p\theta$	is the per unit rotor speed, in electrical radians per unit time (rotor angular speed in mechanical radians per second times pole pairs divided by base angular frequency)

In general, these equations can be solved only by direct numerical integration since they are nonlinear due to the terms containing the product of flux linkage "ψ" and rotor speed "$p\theta$" and the product of "ψ" and "i" in Equation (6). If, however, the acceleration rate of the rotor is sufficiently slow during the starting period and the pulsating torque component can be neglected, the assumption of constant speed is very nearly valid. These conditions, and hence the assumption of "quasi-steady state," are true for all but very small machines. Thus, Equation (1) through Equation (5) may be solved using techniques appropriate for linear systems of equations, in which case, Equation (6) continues to yield the correct torque. Phasor analysis is typically applied to the solution of such circuit problems. The equivalent circuit is shown in Figure 1.

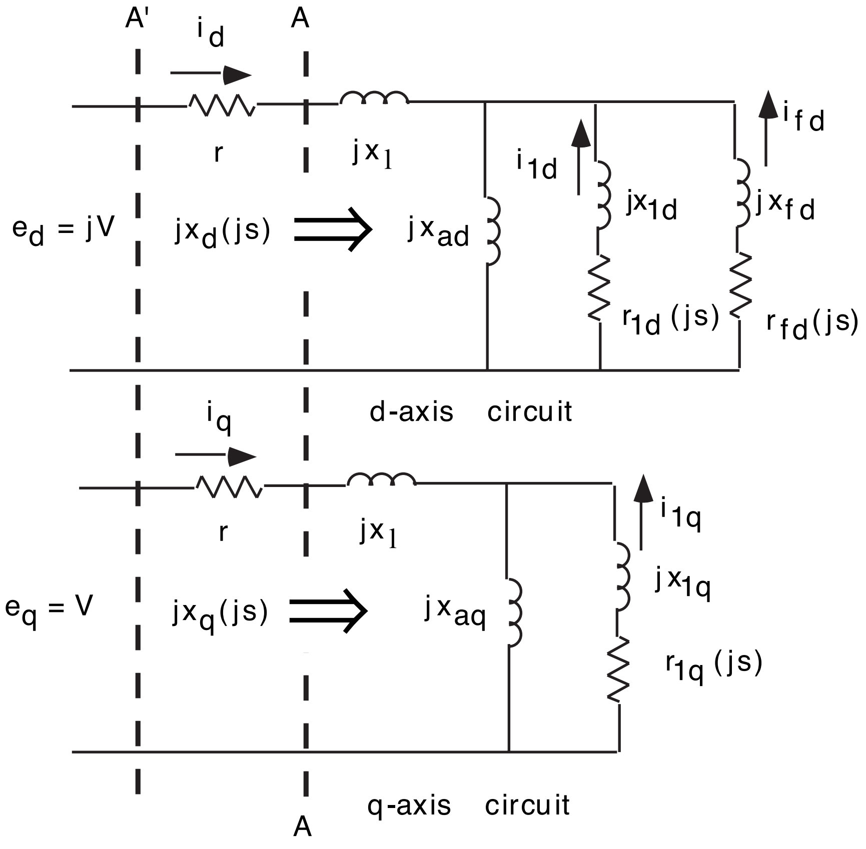

Figure 1— Steady-state d–q axis equivalent circuit of a salient pole synchronous motor operating asynchronously

In Figure 1, the following definitions for the machine parameters apply:

r	is the per unit armature resistance per phase (including external resistance)
x_l	is the per unit armature leakage reactance per phase (including external reactance)
r_{1d}	is the per unit direct axis amortisseur resistance (referred to the stator winding)
x_{1d}	is the per unit direct axis amortisseur leakage reactance (referred to the stator winding)
r_{fd}	is the per unit field resistance (including discharge resistance during starting, and referred to the stator winding)
x_{fd}	is the per unit field leakage reactance (referred to the stator winding)
x_{ad}	is the per unit direct axis mutual reactance
x_{aq}	is the per unit quadrature axis mutual reactance
x_{1q}	is the per unit quadrature axis amortisseur leakage reactance (referred to stator winding)
r_{1q}	is the per unit quadrature axis amortisseur resistance (referred to the stator winding)
s	is the slip angular frequency as per unit of base angular frequency
p	is the derivative operator $d/d(\omega t)$, where ω is the base frequency and t is time
$p\theta$	is the per unit rotor speed, in electrical radians per unit time (rotor angular speed, in mechanical radians per second, times pole pairs divided by base angular frequency)
V	is the per unit infinite bus voltage
$x_d(js)$	is the per unit operational reactance of the d-axis circuit
$x_q(js)$	is the per unit operational reactance of the q-axis circuit

Measurement of the parameters shown in Figure 1 can be established by appropriate testing (see IEEE Std 115-1995[2]). All parameters are assumed to be in per unit. Definitions of the choice of per unit systems can be found in the IEEE Standard Dictionary of Electrical and Electronics Terms [B8]. In particular, the base voltage and power used for determination of the per unit parameters are

$$Z_{\text{base}} = \frac{V_{\text{base}}^2}{P_{\text{base}}} \tag{7}$$

where

V_{base}	is the rms line-to-line voltage, in volts
P_{base}	is the base kVA apparent power input of the motor, in volt-amperes
Z_{base}	is the base impedance per phase (line to neutral), in ohms

If the machine is not connected to an infinite bus, the resistance r and the leakage reactance x_l should include the Thevenin equivalent resistance and reactance of the supply (balanced conditions assumed). The voltages applied to the equivalent circuits are equal in magnitude to the per unit infinite bus voltage V. The voltage applied to the d axis leads the voltage applied to the q axis by 90 electrical degrees. The stator currents that flow in the two circuits are represented as phasor currents $\tilde{I}_d$ and $\tilde{I}_q$.

It can be observed that, for simplicity, only one amortisseur circuit has been assumed in each axis. In this case the parameters x_{1d}, x_{1q}, and x_{fd} are constants (for constant stator frequency) and

$$r_{1d}(js) = r_{1d}/s \tag{8}$$

$$r_{1q}(js) = r_{1q}/s \tag{9}$$

$$r_{fd}(js) = r_{fd}/s \tag{10}$$

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[2] Information on references can be found in Clause 2.

If desired, more accurate representations of the damper circuits can be incorporated by adding extra amortisseur circuits. Alternatively, a single amortisseur circuit in each axis can be retained. In this case the parameters r_{1d}, x_{1d}, r_{1q}, and x_{1q} are considered variables (functions of the complex slip *js*) that relate the more complicated circuit to a circuit with one equivalent amortisseur branch in the *d*– and *q*– circuits (see Owen et al. [B14]). In order to maintain the simplicity of the equivalent circuit, this second approach is recommended. For example, the effects of deep bars, discrete bars, and solid iron can be modeled by this technique. In such cases it is suggested that these equivalent circuit parameters be obtained from the manufacturer in tabular form as a function of slip frequency.

The close analogy of the equivalent circuits to the conventional per phase equivalent circuit of an induction machine can be noted. However, in this case, the circuits in the *d* and *q* axes can not be reduced to a single "per phase" circuit due to the asymmetry of the parameters in the two circuits. The per unit quantities $x_d\,(js)$ and $x_q\,(js)$ are traditionally termed the *operational reactances* of the *d*- and *q*-axis circuits and correspond to the Thevenin impedances looking into the circuit from the point A-A in Figure 1. Note that the two operational reactances do not include the effect of the stator resistance *r,* but do include the rotor resistance and, therefore, have a real as well as an imaginary part. The values of $jx_d\,(js)$ and $jx_q\,(js)$ are explicitly given in Annex B for the case of a single amortisseur winding in each magnetic axis.

5. Calculation of pulsating and average electromagnetic torque

The calculation of instantaneous per unit torque can be computed as the instantaneous per unit power transferred across the air gap of Figure 1, in much the same manner as for an induction machine. A derivation of simplified equations for both the double slip-frequency torque pulsation and the average torque is given in Owen et al. [B14]. It is shown therein that, if the stator resistance is neglected and if quasi-steady state is assumed, the pulsating torque amplitude, T_{puls}, at any time during the acceleration period, can be compactly expressed in per unit as

$$T_{\text{puls}} = \frac{1}{2}\left|\tilde{I}_d\right|\left|\tilde{I}_q\right|\left|x_d(js) - x_q(js)\right| \tag{11}$$

where

$\tilde{I}_d$ is the direct axis current phasor, in per unit
$\tilde{I}_q$ is the quadrature axis current phasor, in per unit
s is the slip angular frequency, in per unit of base angular frequency
$x_d\,(js)$ is the d-axis operational reactance, in per unit
$x_q\,(js)$ is the q-axis operational reactance, in per unit

Note that the last member of the product is the absolute value of the difference between the complex impedances. It is not the difference between the absolute values of the impedances. An important assumption in the derivation of this equation is that the applied voltages $\tilde{E}_d$ and $\tilde{E}_q$, as well as the stator voltages across the *d*- and *q*-axis circuits at the point A-A, must be in time quadrature (90° out of phase). It is clear that this will be true only if the rotor is symmetrical. Although Equation (11) is only approximately correct for salient pole machines, the error is quite small and the equation is considerably more accurate, particularly around half speed, than if the stator resistance were neglected entirely.

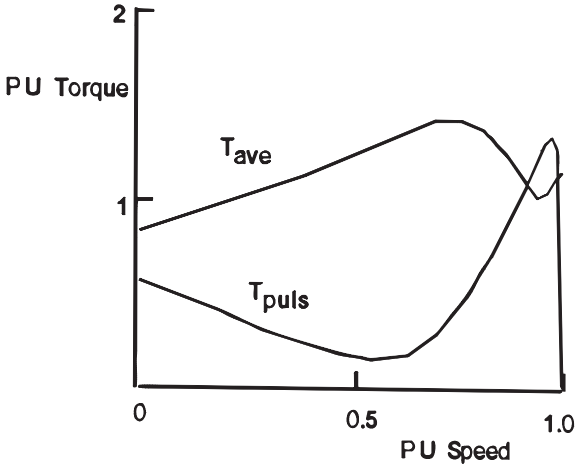
The average component of the electromagnetic torque can also be derived directly from the equivalent circuit. The average torque, T_{ave}, in per unit, is

$$T_{\text{ave}} = \frac{1}{2}\text{Im}[\tilde{I}_d x_d(js)\tilde{I}_q^* - \tilde{I}_q x_q(js)\tilde{I}_d^*] \tag{12}$$

The superscript “*” indicates “conjugate” and “Im” indicates the “imaginary part of ” the bracketed complex quantity.

Figure 2 shows an example calculation of both pulsating and average torques using Equation (11) and Equation (12). The curve, together with the parameters used for this calculation, are of a 3360 kW (4500 hp) machine taken from Owen et al. [B14]. This machine has the following ratings:

— Base power is 4500 hp.
— Base apparent power is 3435 kVA.
— Base rms line-to-line voltage is 4.00 kV.
— Base rotor speed is 1800 r/min.



$x_l = 0.0932$
$x_{ad} = 1.28$
$x_{aq} = 0.770$
$x_{fd} = 0.1838$
$x_{kd} = 0.096$
$x_{kq} = 0.115$

$r = 0.0051$
$r_{fd} = 0.001$
$r_{kd} = 0.085$
$r_{kq} = 0.032$

Figure 2—Calculated speed-torque characteristics at full voltage of a typical synchronous motor and corresponding parameters

6. Mechanical measurement of electromagnetic air gap torque

The electrical system has a strong influence on the level of pulsation torque developed by a synchronous machine so that, unfortunately, simple mathematical models are not able to accurately predict the torque pulsation levels. Mechanical measurement is the preferred test method. Test methods used to measure air gap torque can be divided into two main categories: mechanical methods and electrical methods. The mechanical methods entail measuring the mechanical state (velocity or acceleration) of an unloaded motor shaft which, in reality, indirectly measures the shaft torque. Alternatively, if the shaft is loaded, the strain between the motor shaft and the load shaft can be measured directly.

For all transient measurements during motor starting, a power system with the ability to regulate the supply frequency and voltage to preset values throughout the duration of the test is necessary.

6.1 Measurement of angular acceleration

The angular acceleration can be related to the instantaneous electromagnetic torque by measurement or calculation of the inertia of the rotor, T_e, using Equation (13), in SI units

$$T_e = \frac{J}{N_{pp}} \frac{d^2\theta}{dt^2} + T_{w+f} \tag{13}$$

where

J	is the mass moment of inertia of the rotor
N_{pp}	is the number of pairs of poles
T_{w+f}	is the torque drop due to windage and friction
θ	is the angular velocity of the rotor, in electrical radians per second
t	is time, in seconds

The effect of windage and friction is minor and can be neglected, unless the machine has an unusually large bearing or fan. Also, it is cautioned that these tests are normally conducted at reduced voltage and the results must be corrected to full voltage.

The measurement of acceleration can be obtained by piezoelectric accelerometers (see Mruk et al. [B12]). However, the piezoelectric accelerometer has two important limitations: transverse sensitivity and poor, low-frequency response. Transverse sensitivity corresponds to the problem of accelerometer errors due to forces normal to the axis of rotation. Although typical manufacturer specifications for transverse sensitivity range from $< 1\%$ to 5%, improper mounting geometry can significantly increase this error. In addition, the lower limit of frequency response for piezoelectric accelerometers is typically a few hertz, making them unsuitable for direct measurement of the average torque component.

For other angular accelerometers, such as those that are closed loop, dc-operated force balance transducers may be more suitable, provided that they are chosen with the proper acceleration range and nominal natural frequency. They are delicate devices that must be handled carefully, mounted accurately on the shaft center, and not subjected to any significant axial acceleration. Low pass filtering (≤ 200 Hz) of the output signal is normally required. Attenuation of the signal due to the filtering action should be carefully compensated.

The instantaneous synchronous-motor air gap torque can be determined by monitoring the output from a shaft-mounted angular accelerometer during an acceleration test from standstill through to synchronous speed. Figure 3 shows the schematic arrangement for the typical instrumentation and recording equipment used for an acceleration test with an angular accelerometer. Inclusion of the optional starting breaker shown in Figure 3 enables the preset stator-terminal voltage to be observed along with the zero-acceleration reference prior to the test. With a multichannel analog or digital recorder, additional signals, including line volts, inrush current, rotor speed, and electrical input power, can be captured simultaneously. The frequency response of the recording system must be flat from 0 Hz to beyond at least twice the locked-rotor slip frequency. Calibration signals should be recorded following the test for all signal parameters so that a direct correlation exists between the magnitude of the test signals and the calibrated reference.

A compressed version of the typical data recorded during an acceleration test on a 17 900 kW (24 000 hp) motor is shown in Figure 4. Figure 5 shows an expanded portion of the data in the vicinity of 70% rated speed. The angular acceleration data, $d^2\theta/dt^2/(N_{pp})$, is related to the air gap torque in accordance with Equation (13). The mass moment of inertia of the rotor, J, can be established by calculation or by measurement. A sample analysis of the angular acceleration data describing the derivation of motor per unit air gap torque from the angular accelerometer trace is given in Annex A.

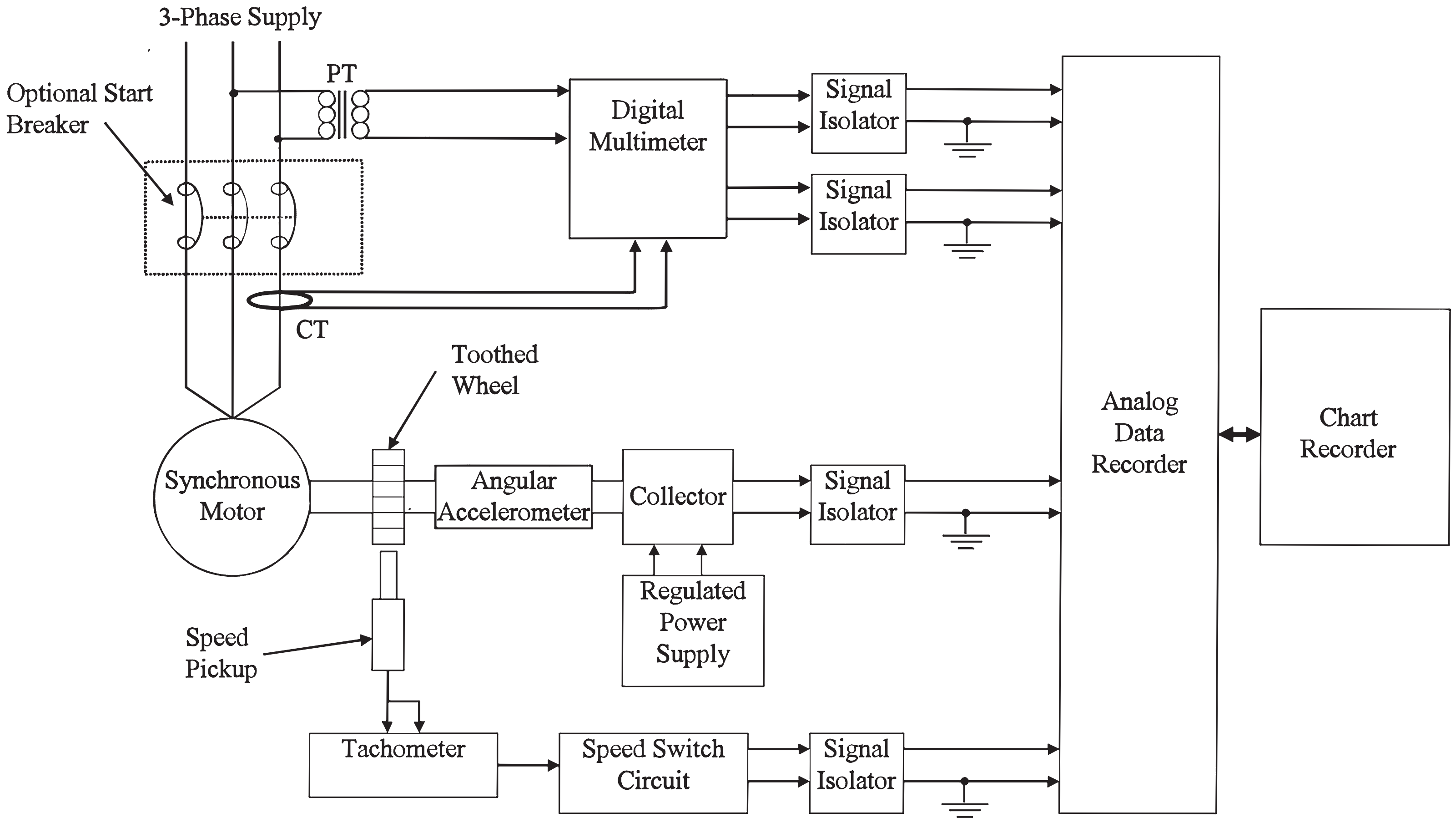


Figure 3— Schematic of acceleration torque test using angular accelerometer

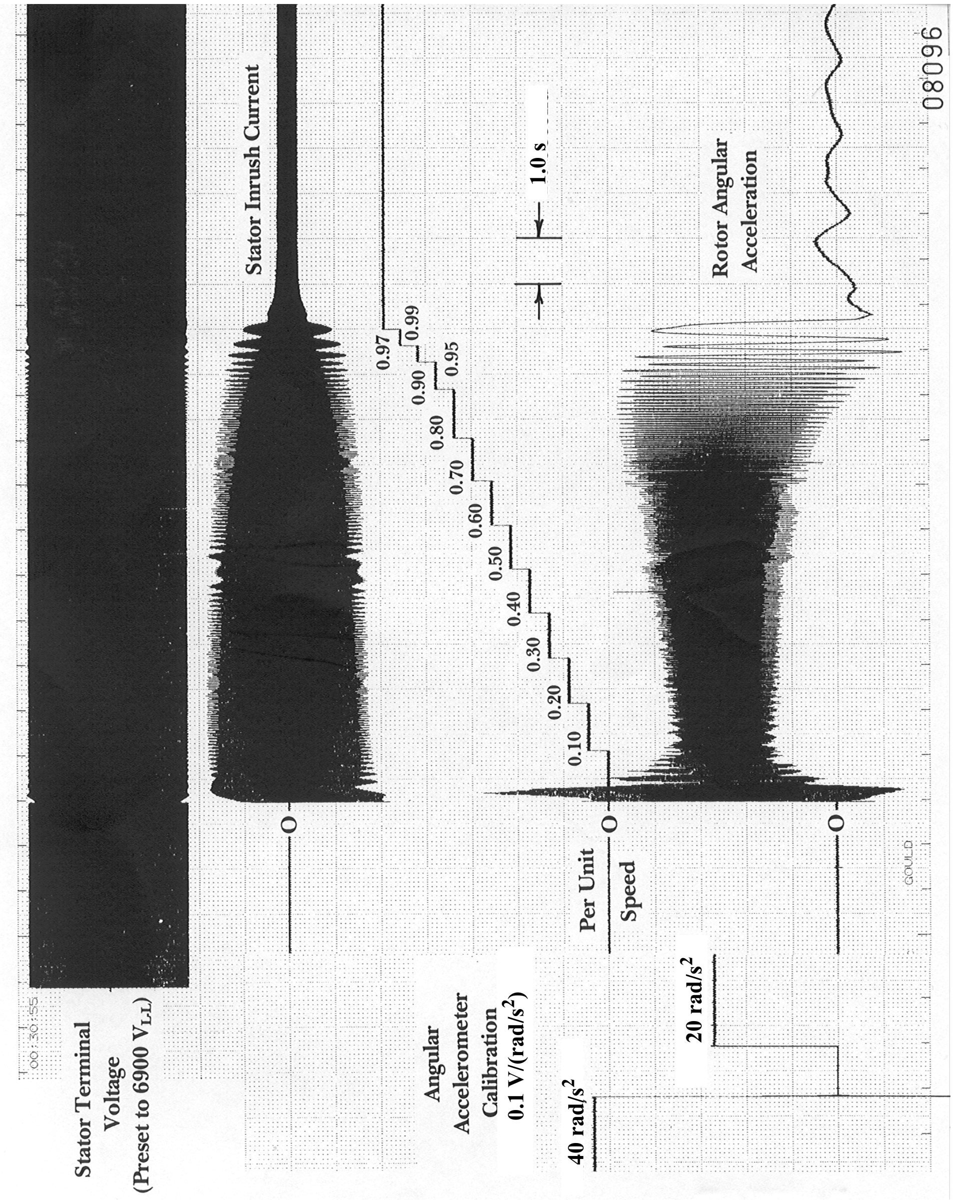


Figure 4— Data acquired during acceleration torque test

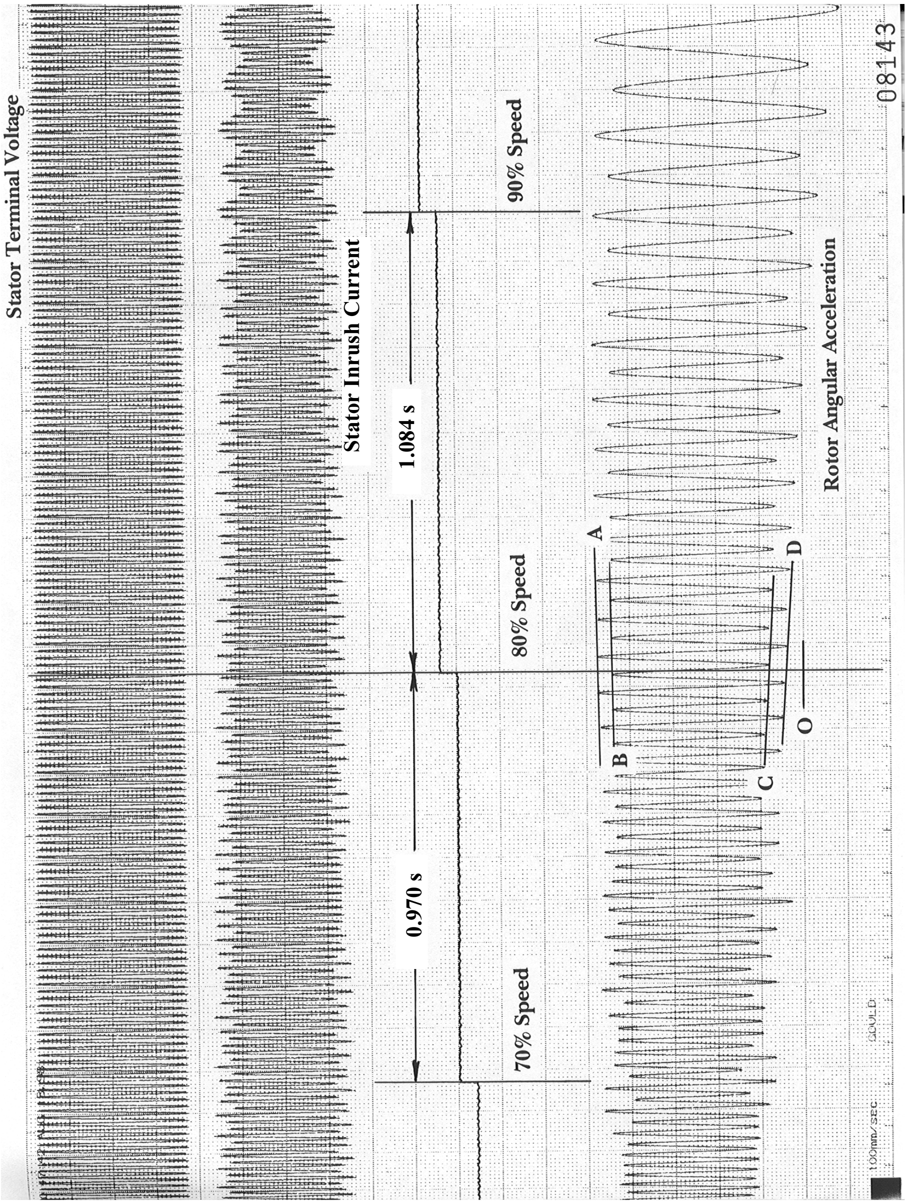


Figure 5— Section of expanded data from acceleration torque test

An independent correlation of the average air gap torque, determined by the angular accelerometer, can be made by comparing each value at a particular speed with the torque derived from the time required for the rotor to accelerate through a finite speed change. An example of this correlation technique is also included in Annex A.

Figure 6 illustrates the measured average and pulsating per unit air gap torque values along with inrush current as a function of speed, adjusted to rated voltage. Data recorded during the initial switching transient at the start of the test, including the effect of a momentary power supply voltage dip and line frequency torque transient, are usually ignored. Zero-speed-starting characteristics can be determined by extrapolation of the data from approximately 10% speed.

6.2 Measurement of rotational velocity

This category of tests includes all devices that nominally produce an output signal proportional to the instantaneous shaft velocity. Theoretically, the angular velocity signal can be differentiated to provide a shaft angular acceleration signal from which the air gap torque can be determined. During the starting period, the pulsating component of air gap torque produces incremental speed variations about the average motor speed. The resolution required to detect the incremental angular speed variations is found by examining the ratio of speed variation to average angular speed during asynchronous operation of typical high-speed synchronous motors. A resolution on the order of one part in one thousand is required to detect the speed variations. Significantly finer resolutions are required to allow differentiation of the angular velocity signal needed to generate an angular acceleration signal. While success has been achieved in measuring instantaneous angular acceleration by this method, it is not recommended over the accelerometer method.

6.3 Torque shaft method

This method uses a special shaft termed a *torque shaft,* which is attached between the motor and its load, for measuring the torque transmitted from the motor to the load on an instantaneous basis. Telemetry to measure the peripheral strain is installed around the shaft. The strain measurement is calibrated to an equivalent torque prior to its installation for the test. The system is rather delicate, but when used properly, gives results over a bandwidth much wider than the torque pulsation frequencies of interest. The measured strain, in SI units, can then be related to the instantaneous electromagnetic torque using Equation (14).

$$T_{\text{strain}} = \left[T_e - T_{w+f} - \frac{J}{N_{pp}}\frac{d^2\theta}{dt^2}\right] - \left[\frac{J_{\text{load}}}{N_{pp}}\frac{d^2\theta_{\text{load}}}{dt^2} + T_{\text{load}}\left(\theta_{\text{load}}, \frac{d\theta_{\text{load}}}{dt}\right)\right] \tag{14}$$

where

J	is the mass moment of inertia of the rotor
N_{pp}	is the number of pairs of poles
t	is the time
T_e	is the air gap torque
T_{w+f}	is the torque drop due to windage and friction
$d\theta/dt$	is the angular velocity of the rotor
load	as a subscript denotes the components of inertia, torque, and electrical angular rotation on the load side of the torque shaft

The effect of windage and friction is again minor, and can be neglected unless the machine has an unusually large bearing or fan. When the connected load is zero, the strain gauge reading becomes zero and the equation reverts to Equation (13).

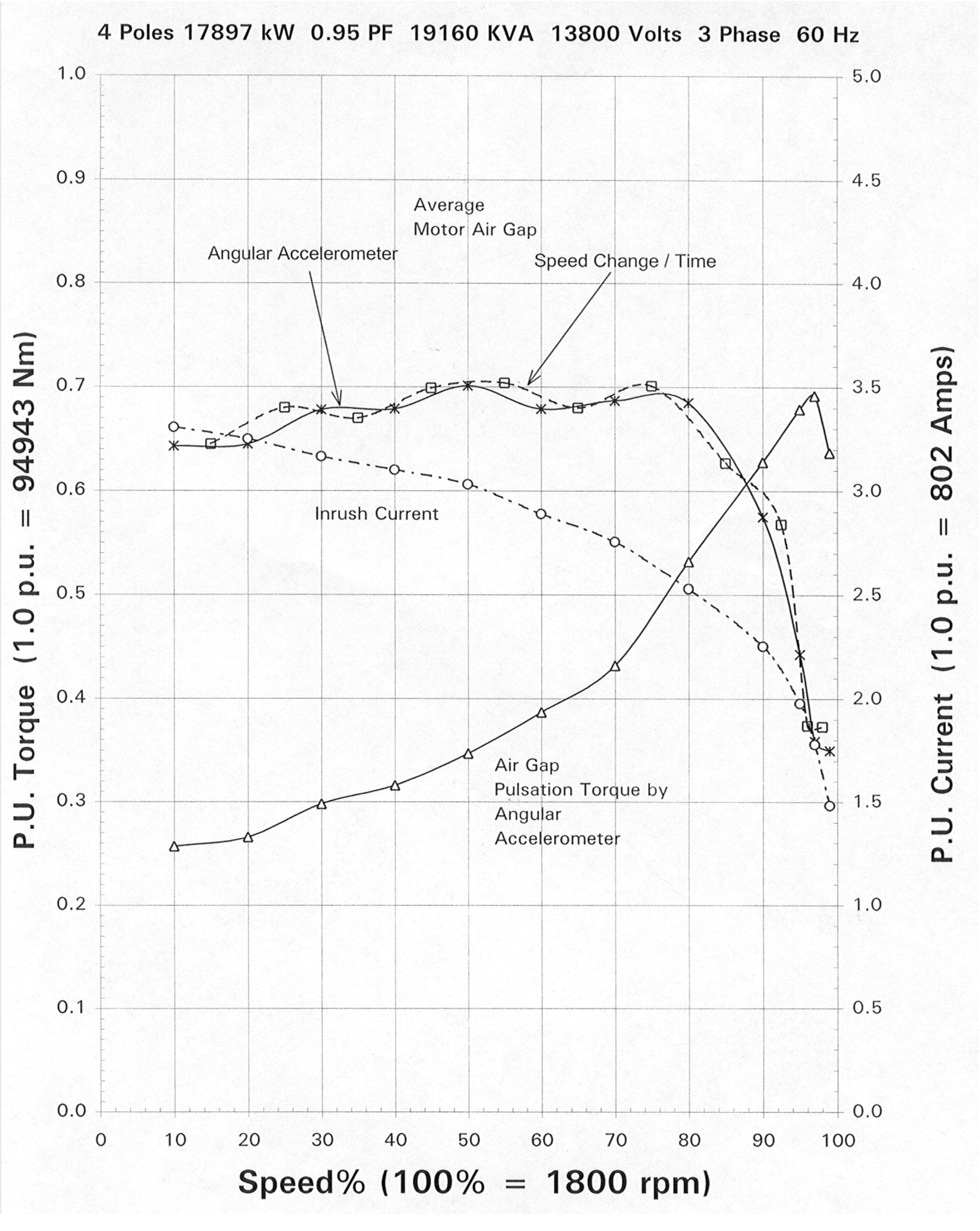


Figure 6—Final result of acceleration torque test

In general, the load varies both as a function of rotational position and rotational speed. If the load characteristics are completely known as a function of these parameters, the air gap torque, T_e, can be solved using Equation (14). However, only in exceptional cases (e.g., when the load is simply an inertial load) can this calculation be completed accurately.

Clause 3 describes the main interest in establishing the air gap pulsation torque relative to the average air gap torque during starting of synchronous motors. Many coupled synchronous motor drives have a torsional natural frequency in the order of 20 Hz. During acceleration of the drive with a 60 Hz system, this natural frequency would be excited at a speed of 83%. Use of a torque shaft, which is typically less stiff then the machine shaft, could easily exacerbate the resonance problem, resulting in its rupture. In exceptional cases, a “special” stiff shaft could be used on very small applications to achieve a torsional natural frequency in excess of 120 Hz, but this approach would not be practical on large, high-inertia drives where the need is greatest. Hence, this method is not recommended over the accelerometer method.

7. Electrical methods for measuring electromagnetic air gap torque

Essentially four electrical methods exist for measuring torque from electrical quantities. In each case, use of voltage and current transducers are required to maintain high-accuracy readings of not only amplitude, but also phase. These methods all utilize the equivalent circuit of Figure 1 and Equation (1) through Equation (6) to calculate the torque from its definition in terms of electrical quantities, Equation (6).

7.1 Search coil method

While difficult to install, the use of search coils represents probably the most accurate method of measuring torque pulsations. In this case small, special-purpose windings, typically made of teflon-coated copper wire, are placed in the air gap of the machine. The voltages induced in these windings are integrated to provide a measure of the flux linkage. When multiplied by suitable currents, the result is proportional to the electromagnetic torque. Alternatively, Hall probes can be substituted for the search coils, in which case the air gap flux is measured directly. The use of Hall probes in the air gap of electric machines unfortunately has severe practical limitations. There is the possibility that these probes can be damaged during installation in the air gap or in the course of measurement. Since the probes are placed directly on the armature winding, their performance can be degraded as the temperature increases due to thermal drift of the sensor.

Kamerbeek [B9] has shown that if hysteresis is neglected and if the magnetic field can be considered as a state function, then the torque acting on the shaft, T_e, in SI units, can be expressed as the integral equation

$$T_e = \int_A R B_r H_\phi dA \tag{15}$$

where

A is the area of the cylindrical surface described by the air gap
B_r is the radial component of air gap magnetic flux density
H_ϕ is the tangential component of magnetic field intensity
R is the radius from the shaft center line to the surface of the stator core facing the air gap

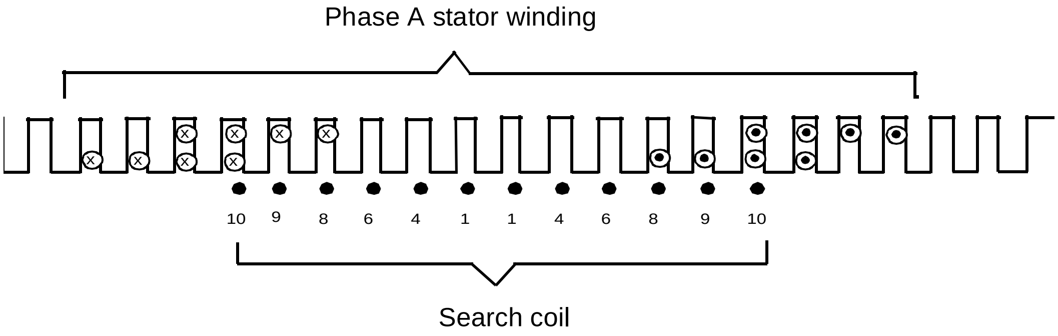
The leakage flux out of the ends of the machine is neglected. Assuming that no eddy currents or permanent magnetization exists in the stator iron and that saliency exists only on the rotor, this concept can be shown to lead to, in consistent (e.g., SI) units (see Holt [B7])

$$T_e = \sqrt{3} \cdot N_{pp} \cdot k_w\left(i_c\int_0^t v_{mb}dt - i_b\int_0^t v_{mc}dt\right) \tag{16}$$

where

k_w	is equal to the effective number of turns of the armature winding divided by the effective number of turns of the search winding, corresponding in each case to the fundamental component of air gap flux density (see Ojo et al. [B13])
N_{pp}	is the number of pole pairs of the machine
v_{mb} and v_{mc}	correspond to the induced voltages in a pair of search coils located as concentric with the magnetic axes of the *b* and *c* phases, as illustrated in Figure 7
i_b and i_c	are the currents in phases *b* and *c*, respectively

A treatment of the design of a search coil is given by Lipo [B10].



NOTE—The numbers below the coil sides of the search coil denote the number of turns.

Figure 7— Developed sketch of stator winding and search coil

While Equation (16) is valid for both star- and delta-connected machines, it should be noted that for Equation (16) to be valid, the machine must be ungrounded (i.e., no zero-sequence component should be present). Because of the cost of mounting the search coils, this method is recommended only where maximum accuracy is required.

7.2 Measurement of electromagnetic torque using terminal voltage and current sensing

Starting from Equation (6), it is shown in Ojo et al. [B13] that the instantaneous torque in per unit can be expressed exactly (consistent with Park's Equations) as

$$T_e = \frac{2}{3\sqrt{3}}\{(i_a - i_b)\int[v_{ca} - r(i_c - i_a)]dt - (i_c - i_a)\int[v_{ab} - r(i_a - i_b)]dt\} \tag{17}$$

where

i_a, i_b, i_c	are the currents in phases *a, b,* and *c,* respectively
v_{ab}	is the voltage of line *a* with respect to line *b*
v_{ca}	is the voltage of line *c* with respect to line *a*
r	is the resistance per phase of the armature

or, alternatively, in SI units, as

$$T_e = \frac{N_{pp}}{\sqrt{3}}\{(i_a - i_b)\int[v_{ca} - r(i_c - i_a)]dt - (i_c - i_a)\int[v_{ab} - r(i_a - i_b)]dt\} \tag{18}$$

Hence, the torque can be computed using only two line-to-line voltages and two current measurements. In Ojo et al. [B13] this method has been termed the volt-second ampere (VSA) method. It should be noted that the current transducers should be capable of sensing dc currents, because during acceleration, the negative sequence currents on the rotor produce a variable frequency component that passes through zero frequency at half speed. The measurement requires an estimate of the armature resistance, which can be determined by averaging the results of a dc measurement just before, and another just after an acceleration test.

In machines above 746 kW (1000 hp), the effect of stator resistance can be considered as negligible. In this case Equation (18) reduces to

$$T_e \approx \frac{N_{pp}}{\sqrt{3}}[(i_a - i_b)\int v_{ca}dt - (i_c - i_a)\int v_{ab}dt] \tag{19}$$

This equation has been termed the modified volt-second ampere (MVSA) method. Equation (18) and Equation (19) are most easily implemented by digital acquisition of the voltage and current measurements and the integration and multiplication implied in Equation (18) and Equation (19) are implemented in digital form either by use of a digital oscilloscope or a data acquisition system. Alternatively, the integration can be carried out in analog form and the equation solved by using a digital wattmeter (see Turgel [B17], Clarke and Stockton [B1], Dix [B4], Stenbakken [B16], and Corney and Pullman [B3]).

Using the VSA method, the unit to be tested is connected electrically as shown in Figure 8. The frequency response of the recording system should be flat from 0 Hz to at least 60 Hz. A speed sensor (dc tachometer or equivalent) with a suitable output should be used to correlate torque as a function of speed. The recording system must be calibrated to maintain accuracy.

7.3 Measurement of electromagnetic torque using input power sensing

The concept of "synchronous watts" (see Mayer and Owen [B11]) provides another means for calculating electromagnetic torque. In this case the power transferred across the air gap is divided by synchronous speed to form "synchronous watts," i.e., approximately, in per unit, as

$$T_e \approx P_{ag} \tag{20}$$

where P_{ag} is the power transferred across the air gap from stator to rotor.

It can be shown that if the voltage at the air gap of the machine is purely sinusoidal, of excitation frequency only, then in per unit Equation (20) is, in fact, an equality (see Gillard [B5]). In terms of terminal quantities, Equation (20) can be written as

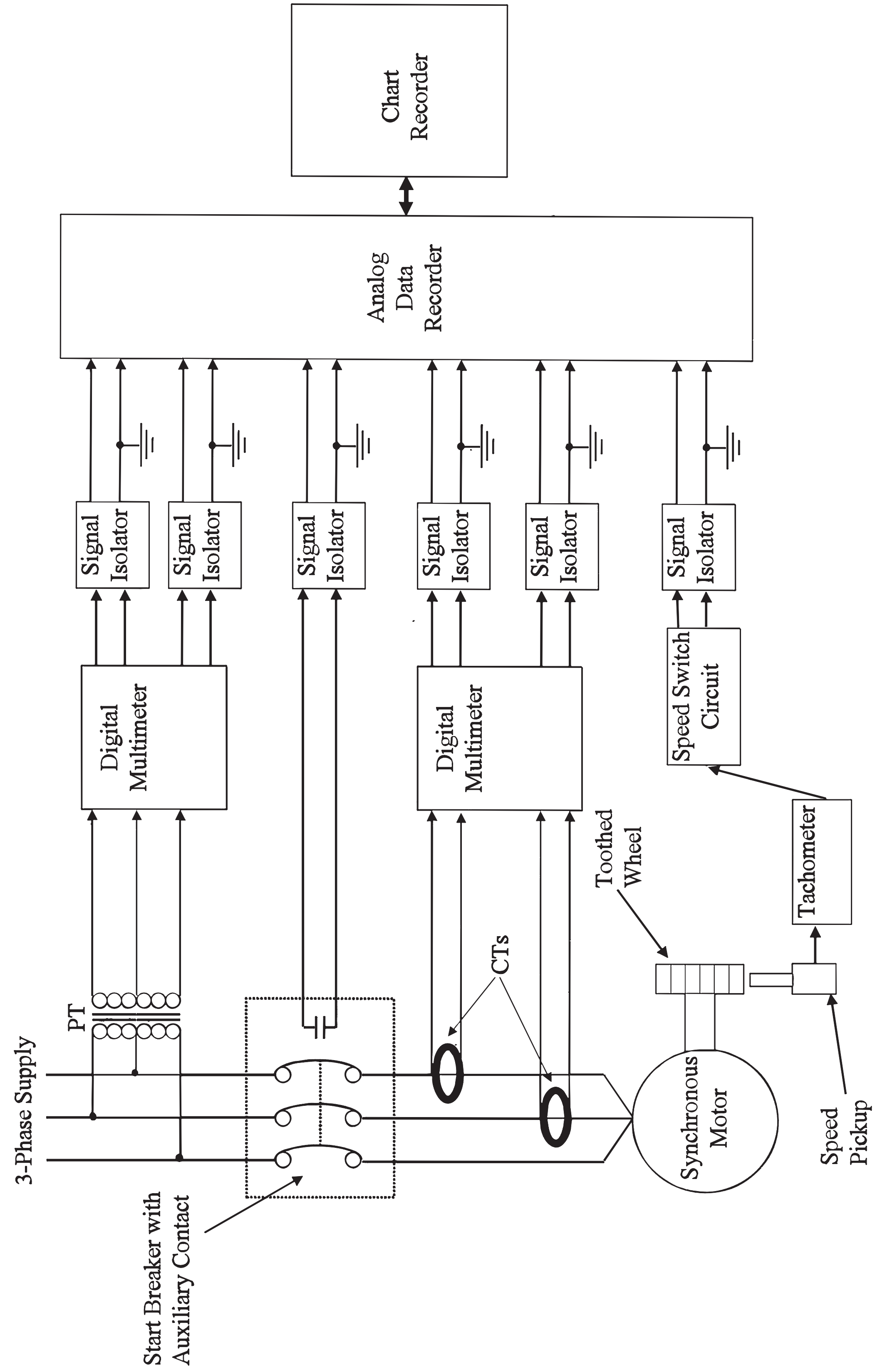


Figure 8—Analog data recording setup for VSA method

$$T_e \approx (P_{\text{in}} - P_{\text{loss}} - P_{\text{leak}}) \tag{21}$$

where

P_{in} is the power flow into the stator terminals
P_{loss} the instantaneous copper losses
P_{leak} is the instantaneous time rate of change of magnetic field energy stored in the stator leakage magnetic field

In consistent physical units (SI, cgs, etc.) Equation (21) becomes

$$T_e = \frac{N_{pp}}{\omega}(P_{\text{in}} - P_{\text{loss}} - P_{\text{leak}}) \tag{22}$$

where ω is the angular frequency of the applied voltages.

The quantities P_{loss} and P_{leak} can be neglected for large machines, resulting in the following simple expression for measuring instantaneous torque:

$$T_e = \left(\frac{N_{pp}}{\omega}\right)P_{\text{in}} \tag{23}$$

This approach is somewhat less accurate than Equation (19) since unbalanced current flow in the *d*– and *q*–axes of the rotor, due to rotor asymmetry, causes the stator currents to have a negative sequence component at source frequency minus twice the slip frequency, even though the supply voltage is purely sinusoidal. This results in errors that can be substantial near the half speed where George's effect takes place (see Mayer and Owen [B11]). However, in practice, for large machines, Equation (23) yields almost identical results when compared with the MVSA method using Equation (19). A block diagram of the setup for the input power method is given in Figure 9. Figure 10 shows a comparison of the VSA, input power, and accelerometer methods respectively. It can be noted that the results for the first two methods are nearly the same while the accelerometer indicates less of a half-speed dip than that of the other two methods.

7.4 Measurement of torque using electrical power input and speed sensing

The relationship between the electrical power absorbed by a synchronous motor and instantaneous mechanical power output can also be derived from Park's Equations (see Holt [B7]). If Park's stator voltage equations for the *d* and *q* axes, Equation (1) and Equation (2), are multiplied by the stator currents that flow in the respective circuits, the following expression for instantaneous power flow into the machine in per unit (P_{in}) is obtained:

$$P_{\text{in}} = e_d i_d + e_q i_q \tag{24}$$

where

e_d, e_q are the direct axis and quadrature stator voltages, in per unit, respectively
i_d, i_q are the direct axis and quadrature axis stator voltages, in per unit, respectively

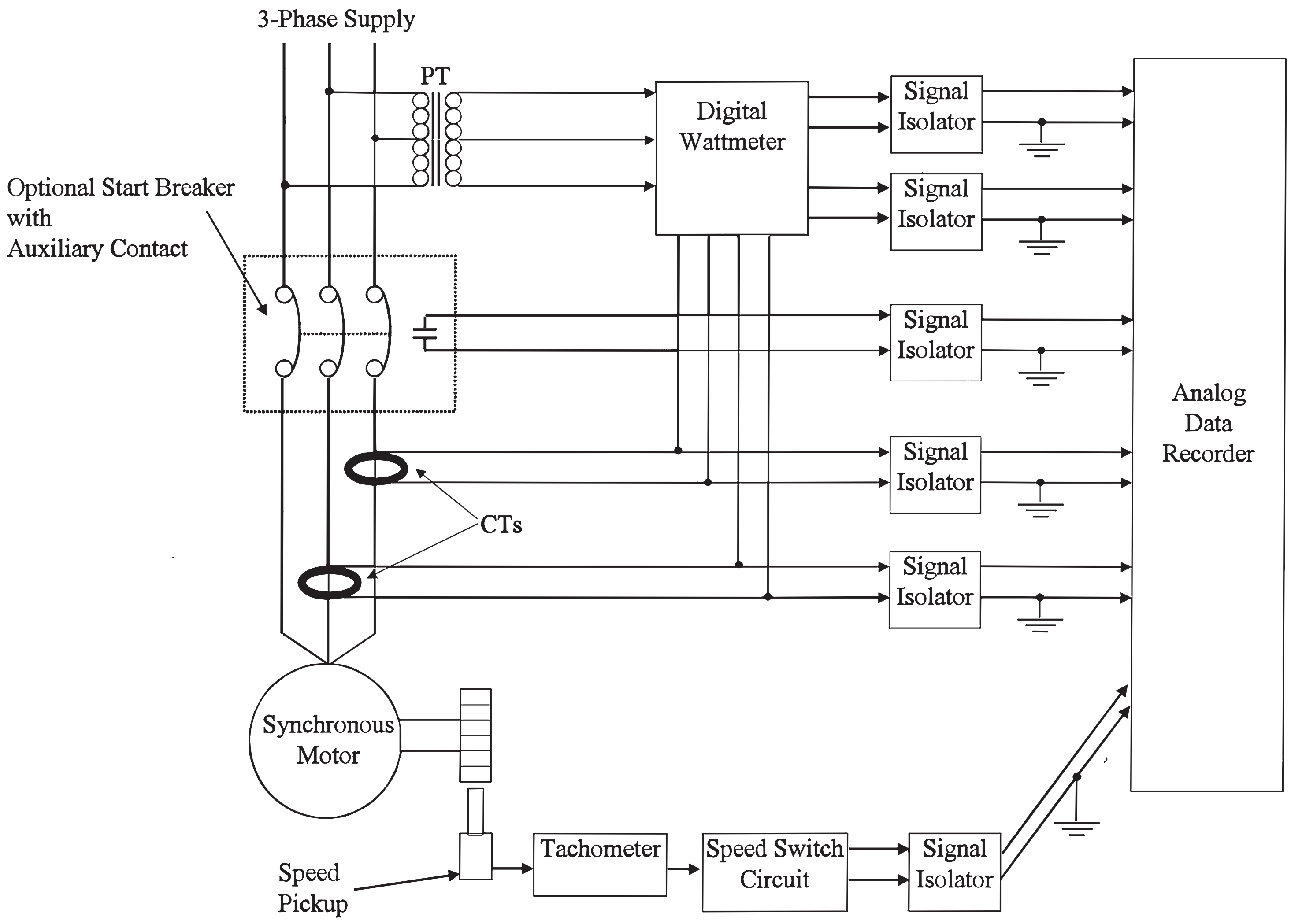
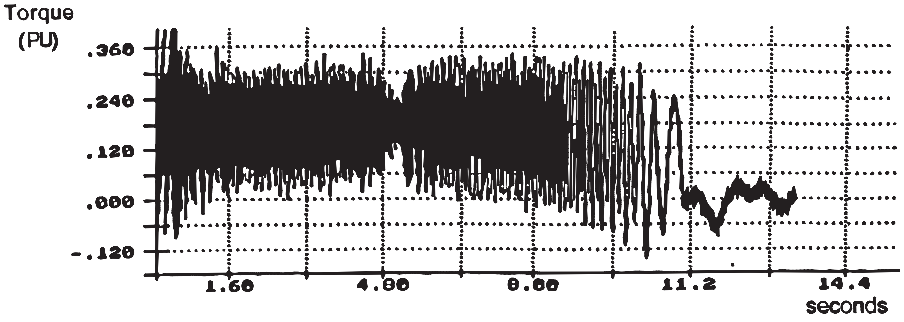
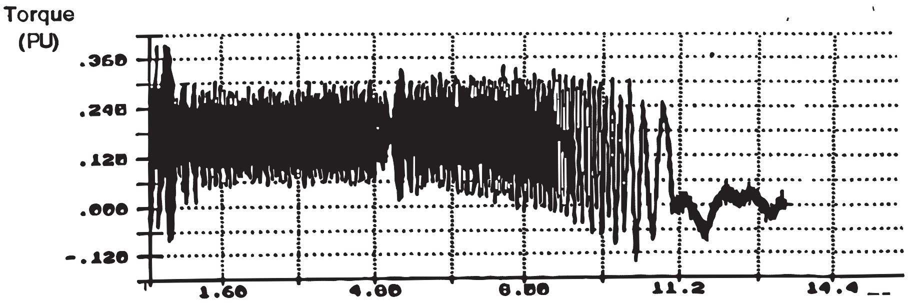


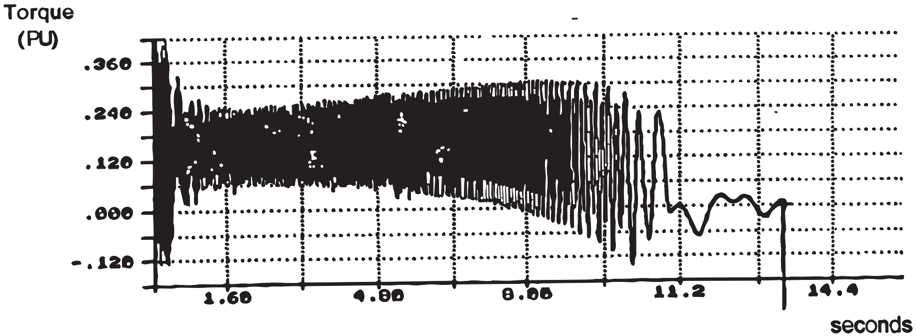
Figure 9—Analog data recording setup for synchronous watts method



a) PU Torque by VSA Method



b) PU Torque by Input Power Method



c) PU Torque by Accelerometer Method

Source: Gillard [B5].

Figure 10— Comparison of experimental torque measurements for VSA, input power, and accelerometer methods for the same 10 440 kW (14 000 hp) synchronous motor

The input power is clearly

$$P_{\text{in}} = P_{\text{loss}} + P_{\text{mag}} + P_{\text{out}} \tag{25}$$

where

P_{loss} is the instantaneous stator copper loss, in watts
P_{mag} is the instantaneous time rate of change of magnetic field energy stored in the stator total magnetic field, in joules per second
P_{out} is the shaft output power, in watts

When Equation (24) is solved, the equations for the various time rates of change of energy components are given as follows:

$$P_{\text{loss}} = r(i_d^2 + i_q^2) \tag{26}$$

$$P_{\text{mag}} = i_d p\Psi_d + i_q p\Psi_q \tag{27}$$

$$P_{\text{out}} = p\theta(i_q\Psi_d - i_d\Psi_q) \tag{28}$$

Comparison of Equation (28) with Equation (6) reveals that the electromagnetic torque is, in per unit,

$$T_e = \frac{1}{p\theta}P_{\text{out}} \tag{29}$$

or without approximation

$$T_e = \frac{1}{p\theta}(P_{\text{in}} - P_{\text{mag}} - P_{\text{loss}}) \tag{30}$$

or in SI units

$$T_e = \frac{1}{\omega_{rm}}(P_{\text{in}} - P_{\text{mag}} - P_{\text{loss}}) \tag{31}$$

where ω_{rm} is the actual mechanical angular speed of the rotor, in radians per second.

Equation (31) indicates that the instantaneous electromagnetic air gap torque can be obtained by subtracting the ohmic losses and time rate of change of stator magnetic field energy from the input power, then dividing by rotor speed. Again, the second two terms in the parentheses on the right-hand side of Equation (31) might be neglected. However, during acceleration, the total stator magnetic field energy changes relatively rapidly making the result considerably less accurate than Equation (23), rendering this approach impractical. Also, the quantity is indeterminate at zero speed due to the division by zero. Since the method is much less accurate than the VSA, MVSA, or synchronous watt techniques, this method is not recommended.

Annex A

(informative)

Sample calculation of per unit air gap torque

The following is a sample calculation based on data at 80% rated speed from Figure 5 illustrating the derivation of per unit air gap torque from the angular acceleration trace.

From Figure 5 at 80% speed:

Point O represents the zero level for the accelerometer signal

Distance OA = 47.2 div

Distance OB = 43.9 div

Distance OC = 7.8 div

Distance OD = 3.9 div

Mean distance from the “O” reference (proportional to average torque)

$$= (47.2 + 43.9 + 7.8 + 3.9) / 4$$

$$= 25.70 \text{ div}$$

Average deviation from the mean distance (proportional to pulsation torque)

$$= [(47.2 - 25.7) + (43.9 - 25.7) + (25.7 - 7.8) + (25.7 - 3.9)] / 4$$

$$= 19.95 \text{ div}$$

From calibration trace, 27.0 div = 20 rad/s^2

or 1 rad/s^2 = 0.74 div

Rotor mass moment of inertia (J) = 855 kg·m^2 (20 300 lb·ft^2)

Rated (*1 PU*) *torque* = 94 900 N·m (70 000 lbf·ft)

From Equation (13), ignoring the windage and friction torque component

$$\textit{Per unit torque} = J \times \alpha / (\textit{Rated torque})$$

where α is the angular acceleration, in mechanical radians per second squared.

$$\textit{Per unit average air gap torque} = 855.5 \text{ kg}\cdot\text{m}^2 \, [25.7 \text{ div} \times 0.74 \, (\text{rad/s}^2) / \text{div}] / (94\,900 \text{ N}\cdot\text{m})$$

$$\text{or} \quad = 630.4 \text{ lbf}\cdot\text{ft}\cdot\text{s}^2 \, [25.7 \text{ div} \times 0.74 \, (\text{rad/s}^2)/\text{div}]/(70\,000 \text{ lbf}\cdot\text{ft})$$

$$= 0.171 \text{ PU}$$

Per unit pulsation air gap torque = 855.5 kg·m^2 [19.95 div × 0.74 (rad/s^2) / div] / (94 900 N·m)

or = 630.4 lbf·ft s^2 [19.95 div × 0.74 (rad/s^2) / div] / (70 000 lbf·ft)

= 0.133 PU

Adjusting the torque values to rated terminal voltage from the test data at 50% voltage, and assuming that the torque is proportional to the square of the motor terminal voltage

Rated voltage per unit average air gap torque at 80% rated speed = 0.171 × [(13 800)^2 / (6 900)^2]

= 0.685 PU

Rated voltage per unit pulsation air gap torque at 80% rated speed = 0.133 × [(13 800)^2 / (6 900)^2]

= 0.532 PU

Verification of the average air gap torque, $T_{g,ave}$, over a particular speed range can be made in accordance with Equation (32) [Equation (7-5) from IEEE Std 115-1995].

$$T_{g,\,ave} = \frac{10.97 \times 10^{-6} \times J \times rated\ speed \times (dn/dt)}{rated\ motor\ output} \tag{32}$$

where *dn*/*dt* is the rate of change of speed, in rpm per second.

From Figure 5, the times to accelerate from 70% to 80% speed and from 80% to 90% speed are 0.970 s and 1.084 s, respectively. Solving Equation (32) for an average speed of 75%, and adjusting the result to rated terminal voltage

$$T_{g,\,ave} = \frac{10.97 \times 10^{-6} \times 855.5\ \mathrm{kg \cdot m^2} \times 1800\ \mathrm{rpm} \times (180\ \mathrm{rpm}/0.970\ \mathrm{s}) \times [(13.8)^2/(6.9)^2]}{17\ 900\ \mathrm{kW}} = 0.701\ \mathrm{PU}$$

Similarly, at 85% speed

$$T_{g,\,ave} = \frac{10.97 \times 10^{-6} \times 855.5\ \mathrm{kg \cdot m^2} \times 1800\ \mathrm{rpm} \times (180\ \mathrm{rpm}/1.084\ \mathrm{s}) \times [(13.8)^2/(6.9)^2]}{17\ 900\ \mathrm{kW}} = 0.627\ \mathrm{PU}$$

Annex B

(informative)

Operational reactances

When the rotor equivalent *d–q* circuit can be represented by a single amortisseur circuit per axis, the operational reactances can be found from Equation (33) and Equation (34).

$$jx_d(js) = jx_l + \cfrac{1}{\cfrac{1}{jx_{ad}} + \cfrac{1}{\cfrac{r_{1d}}{s} + jx_{1d}} + \cfrac{1}{\cfrac{r_{fd}}{s} + jx_{fd}}} \tag{33}$$

$$jx_d(js) = jx_l + \cfrac{1}{\cfrac{1}{jx_{aq}} + \cfrac{1}{\cfrac{r_{1q}}{s} + jx_{1q}}} \tag{34}$$

where

j	is the complex number $\sqrt{-1}$
x_d	is the per unit direct axis reactance
s	is the slip angular frequency as a per unit of the base angular frequency
x_l	is the per unit armature leakage reactance
x_{ad}	is the per unit direct axis mutual reactance
r_{1d}	is the per unit direct axis amortisseur winding resistance
x_{1d}	is the per unit direct axis amortisseur winding leakage reactance
r_{fd}	is the field winding resistance, in per unit
x_{fd}	is the field winding leakage reactance, in per unit
x_{aq}	is the per unit quadrature axis mutual reactance
r_{1q}	is the per unit quadrature axis amortisseur winding resistance
x_{1q}	is the per unit quadrature axis amortisseur winding leakage reactance

Annex C

(informative)

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